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The non-relativistic limit of Dirac operators with electrostatic and Lorentz scalar δ-shell interactions in $\mathbb{R}^3$

MASTER'S THESIS

to achieve the university degree of

Diplom-Ingenieur

Master's degree programme: Mathematics

submitted to

Graz University of Technology

Supervisor

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Institute of Applied Mathematics

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Graz, June 2020

Abstract

In this master thesis the non-relativistic limit of Dirac operators with electrostatic and Lorentz scalar δ-shell interactions in $\mathbb{R}^3$ is investigated. These interactions appear, for instance, as idealizations in the description of a relativistic quantum particle with spin $1/2$ in the presence of strongly localized external fields. In order to describe δ-shell interactions, we consider the formal differential expression

$$\mathcal{A}_{\eta,\tau} = A_0 + (\eta I_4 + \tau\beta)\,\langle\delta_\Sigma, \cdot\rangle\delta_\Sigma$$

as a singular pertubation of the free Dirac operator A_0. Here, Σ is a compact, closed and C^2-smooth surface in $\mathbb{R}^3$, $\eta, \tau \in \mathbb{R}$ represent the strengths of interaction and $I_4, \beta \in \mathbb{C}^{4\times 4}$ are two matrices. Applying the theory of quasi boundary triples, self-adjoint operators $A_{\eta,\tau}$ can be constructed by encoding the effect of the δ-interactions in form of suitable jump conditions on the interface Σ. These operators are interpreted as realizations of the formal differential expression above.

Subsequently, for $\lambda \in \mathbb{C} \setminus \mathbb{R}$ the non-relativistic limit

$$\big(A_{\eta,\tau} - \big(\lambda + mc^2\big)\big)^{-1} \to \begin{pmatrix} (T_{\eta,\tau} - \lambda)^{-1} & 0 \\ 0 & 0 \end{pmatrix} \quad \text{for } c \to \infty$$

is determined for the resolvent, where $T_{\eta,\tau}$ is a self-adjoint operator. The corresponding convergence analysis and the characterization of the limit operator $T_{\eta,\tau}$ is done separately for the two cases $\eta + \tau \neq 0$ and $\eta + \tau = 0$, as in these the limit operators behave quite differently.

For the parameter combination $\eta + \tau \neq 0$, the limit operator $T_{\eta,\tau}$ turns out to be a Schrödinger operator with a δ-interaction of strength $\eta + \tau$. This indicates that the Dirac operators $A_{\eta,\tau}$ can indeed be regarded as relativistic counterparts of the well studied Schrödinger operators with δ-interactions.

Finally, it is shown that in the case of $\eta + \tau = 0$, the limit operator $T_{\eta,\tau}$ is a Schrödinger operator as well. However, the characterization of the domain of definition yields that, in contrast to the case $\eta + \tau \neq 0$, there are no jump conditions describing δ-interactions but oblique jump conditions.

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