Hagen Neidhardt (1950–2019) – His Work and Legacy

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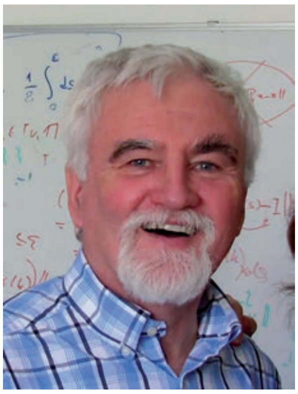


Our friend and co-author, brilliant mathematician Hagen Neidhardt, passed away on 23 March 2019 at the age of 68. Author of more than 150 research papers, Hagen was a world-renowned expert in the areas of functional analysis, operator theory and mathematical physics, where he made a number of highly original contributions.

Hagen Ernst Neidhardt was born on 20 November 1950 in the provincial town of Gefell (Thuringia) in the German Democratic Republic (GDR). His father, Hubertus Neidhardt, was director at the Engineering School for Textile Technology, Reichenbach/Vogtland. His mother Ruth Neidhardt (née Löffler) was a clerk at Vogtlandstoffe VEB Kombinat Wool and Silk Meerane.

Hagen was passionate about mathematics from a very young age. On one occasion he came across a mathematics enyclopedia his parents had been planning to give him as a Christmas present that year, and by the time Christmas came around, it was clear to his parents that he had already worked through the entire book. His whole life beat to the rhythm of mathematics.

School, University, Karl Weierstraß Institute

Neidhardt's special talent and inclination towards mathematics were recognized early on by parents and teachers. Hagen attended a local primary school in Gefell from 1957 to 1967, after which he moved on to advanced high schools in Schleiz and Reichenbach (Vogtland).

At only 17 years of age, Hagen left his childhood home for good. In 1967, he moved to the Faculty of Workers and Peasants (ABF) at Martin Luther University Halle-Wittenberg for two years preparation for studying in the Soviet Union. The Institute for Preparation for Studying Abroad in Halle (Saale) prepared delegated students from all over the GDR for studying abroad. Hagen was preparing for his studies in the USSR at the Leningrad State University (LSU).

Six years later, on 28 February 1975, Hagen graduated with distinction from the Faculty of Physics of the LSU and was awarded the diploma in mathematical physics. His tutor and promotor of his diploma thesis on the *scattering theory* was Professor Mikhail Shlemovich Birman.

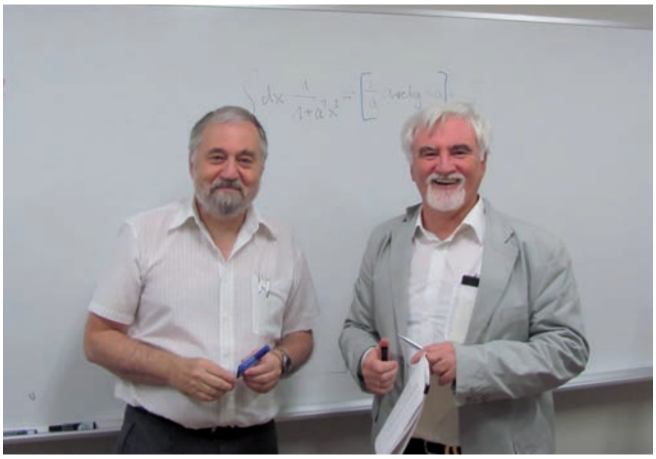
Hagen Neidhardt, Dubna (1989)

This background and his time with Professor Birman, who was then the head of the Leningrad Mathematical Physics Seminar, were decisive for the mathematical orientation of the young Hagen Neidhardt. This became the spectral theory of operators and, in particular, the scattering theory, which were the main topics of the seminar at that time, with participation of Ludwig D. Faddeev, Olga A. Ladyzhenskaya and Boris S. Pavlov.

Almost immediately after his return to the GDR, on 4 March 1975, Hagen took up a junior position in the Karl Weierstraß Institute of Mathematics in Berlin. There he wrote his first research paper in 1976: "Zwei-Raum-Verallgemeinerung des Theorems von Rosenblum und Kato" on spectral analysis of the scattering theory, motivated by one of Birman's publications. This paper appeared in *Mathematische Nachrichten* 84(1978) 195–211 and signified one of the preferred directions of Hagen's scientific interests. He returned to this question in the paper "A nuclear dissipative scattering theory" (*J. Operator Theory* 1985), and then in "A Converse of the Kato-Rosenblum Theorem" in the same journal (1991). The last paper, where he revisited this problem, was published in 2017.

In the meantime, Hagen started to work on his Ph.D. thesis. His supervisor was Professor Hellmut Baumgärtel, a leading expert in the mathematical scattering theory. Although this topic offered enough scope to be worth continuing, Hagen was attracted to another project related to the solution of the non-autonomous Cauchy problem with the help of extension to evolution semigroups. This method was advocated by Howland (1974) and Evans (1976). In "Integration von Evolutionsgleichungen mit Hilfe von Evolutionshalbgruppen" (Dissertation, AdW der DDR, Berlin 1979, defended on 5 April 1979) Hagen generalised this approach to an arbitrary Banach space. The main result, published in "On abstract linear evolution equations. I" (*Mathematische Nachrichten* 1981), proved the one-to-one correspondence between a set of evolution semigroups and strongly continuous solution operators (propagators) for non-autonomous Cauchy problem. The Howland–

Evans–Neidhardt approach is now well known for both parabolic and hyperbolic cases. They were scrutinised by Hagen in two papers, "On abstract linear evolution equations.II" and "III" in 1981–82. He returned to the elucidation of the difficult hyperbolic case versus Schrödinger evolution in "Linear non-autonomous Cauchy problems and evolution semigroups" (*Adv. Diff. Equations* 2009). Hagen liked the evolution semigroups approach to the non-autonomous Cauchy problem and returned to applications of this method many times, in particular, in the framework of product formula approximations for propagators. One of the very last of his papers on this subject: "Convergence rate estimates for Trotter product approximations of solution operators for non-autonomous Cauchy problems" appeared only recently in *Publ. RIMS Kyoto Univ*. 2020.



Pavel Exner and Hagen Neidhardt, Kanazawa (2010).

Joint Institute for Nuclear Research, Dubna

A new chapter in Hagen's scientific evolution opened in the second half of the eighties, when he arrived with his family for a scientific visit (15 September 1986–14 September 1990) to the Joint Institute for Nuclear Research (JINR) in Dubna, USSR. There he first returned to the "spectral shift" problem that was studied in Leningrad by L.S. Koplienko, one of Birman's students. Note that this problem can be traced back to M.G. Krein (1953, 1962), who introduced the terms "spectral shift function" and "trace formula". Let $\{H, H_0\}$ be a pair of self-adjoint operators on a separable Hilbert space $\mathfrak{h}$ which differ by a nuclear operator. M.G. Krein has proved the existence of a summable real function $\xi(\cdot)$ defined on $\mathbb{R}^1$ such that for a certain class of functions $\psi(\cdot)$ the relation

$$\mathrm{Tr}(\psi(H) - \psi(H_0)) = \int_{\mathbb{R}^1} d\lambda\, \xi(\lambda)\, \partial_\lambda \psi(\lambda)$$

holds. The function $\xi(\cdot)$ is called the *spectral shift function* of the pair $\{H, H_0\}$ and the relation itself, the *trace formula*. In his paper "Spectral Shift Function and Hilbert-Schmidt Perturbation: Extensions of Some Work of L.S. Koplienko" (*Mathematische Nachrichten* 138(1):7–25, 1988), Hagen made an important step forward in this problem. Then, in 1987–1990, he generalised the trace formula for non-unitary and non-self-adjoint operators and showed that a summable real spectral shift function can be introduced for a pair of contractions, or dissipative operators, such that the trace formula holds if they differ by an operator which is *slightly* more compact than a trace class operator.

These results constituted a part of Hagen's dissertation of *Doctor scientiarum naturalium* awarded on 30 June 1987 by the Akademie der Wissenschaften der DDR. The formula proved by him is now known as the *Koplienko–Neidhardt Trace Formula*.

Hagen Neidhardt was very friendly with colleagues and always open to new ideas. In the Laboratory of Theoretical Physics of JINR, he was a member of the Mathematical Physics Group headed by Werner Timmermann and then by Pavel Exner. The problems discussed at the group seminar inspired Hagen to new projects.

One of these was motivated by the question from quantum statistical mechanics and brought a new object to his attention: the Gibbs semigroup, i.e., strongly continuous semigroups $\{e^{-tA}\}_{t\geq 0}$ with values in the *-ideal of trace-class operators $\mathscr{C}_1(\mathfrak{h})$ on a separable Hilbert space $\mathfrak{h}$ for $t>0$. The question was whether the well-known strongly convergent *Trotter product formula*

$$\lim_{n\to\infty} \left(e^{-tA/n} e^{-tB/n}\right)^n = e^{-tH}, \qquad t > 0,$$

converges in the *trace-norm* topology to semigroup with some generator H if B is generator of a strongly continuous semigroups. In the paper "The Trotter–Kato product formula for Gibbs semigroups" with V. Zagrebnov (*Commun. Math. Phys*. 1990) this question was answered affirmatively in a general framework of non-exponential Kato functions.

This paper triggered an important long-term research project on the *product formulae* approximations in the trace-norm and the operator-norm topologies for semigroups, unitary groups and for propagators that involved Hagen Neidhardt and his co-authors P. Exner, T. Ichinose and V. Zagrebnov.

In addition to his research work, Hagen actively participated in the life of the community concentrated

Valentin Zagrebnov, Hagen Neidhardt and Jürgen Voigt, QMath7, Prague (1998).

around the mathematical physics group of the Laboratory of Theoretical Physics. He attended conferences that were the beginning of what was later known as the "Mathematical Results in Quantum Physics" (or QMath) series, and helped to organise the third one in 1989, dedicated to the memory of M.G. Krein. He also co-edited this proceedings conference volume of QMath3, which appeared under the title *Order, Disorder and Chaos in Quantum Systems* as Volume 46 in the Birkhäuser series "Operator Theory: Advances and Applications" (Basel 1990).

Back to Berlin

In September 1990, Hagen returned with his family to Berlin. It was not an easy time for them. One of the results of the German "reunification" was the demise of the Karl Weierstraß Institute of Mathematics, with all of its employees having been made redundant. For two years in 1992–1993 he worked at the Technical University of Berlin, and from the 1st of January 1994 to 31 December 1999 he was a research associate at the University of Potsdam. These difficulties neither discouraged him, nor did they reduce his enthusiasm for doing mathematics.

At that time he often visited the Mediterranean University of Marseille-Luminy to continue the collaboration with V. Zagrebnov on the Trotter–Kato product formula and operator-norm convergence. They also started a new project on singular perturbations, regularisation and extension theory; let us quote a few principal papers in this connection: "Towards the right Hamiltonian for singular perturbations via regularization and extension theory" (1996), "Does each symmetric operator have a stability domain?"(1998), "On semibounded restrictions of self-adjoint operators"(1998). These results motivated an important article with P. Exner and V. Zagrebnov, "Potential approximations to δ': an inverse Klauder phenomenon with norm-resolvent convergence"(2001).

During the nineties, Hagen also collaborated closely with J. Brasche on Krein's extension theory and singularly continuous spectrum of self-adjoint extensions, as well as on the inverse spectral theory for self-adjoint extensions. Once more, the research did not consume all of his energy. During his stay at Potsdam University he organised, in collaboration with M. Demuth, P. Exner and V. Zagrebnov, the fifth issue of the QMath conference series in Blossin in the Berlin suburbs, effectively giving the series a new lease of life; he also co-edited the conference proceeds appearing as Volume 70 of the indicated Birkhäuser edition.

Back to the Weierstraß Institute

In January 2000 Hagen Neidhardt succeeded in returning to his mathematical *alma mater*, reborn under the name Weierstraß Institute for Applied Analysis and Stochastic (WIAS). As usual, he was full of plans and enthusiasm.

The Trotter–Kato product formulae activity for semigroups progressed successfully in collaboration with Valentin Zagrebnov, leading to the operator-norm convergence with the rate estimate subsequently extended

Shigetoshi Kuroda and Hagen Neidhardt, Prague (2006)

to symmetrically-normed ideals, and with T. Ichinose, V. Zagrebnov to fractional conditions, "Trotter–Kato product formula and fractional powers of self-adjoint generators" (*J. Funct. Anal.* 2004). A few interesting results together with P. Exner, T. Ichinose and V. Zagrebnov were also established for the unitary case in "Zeno product formula revisited" (*Integral Equations and Operator Theory* 2007) and in "Remarks on the Trotter–Kato product formula for unitary groups" (Integral Equations and Operator Theory 2011).

During one of his visits to Marseille-Luminy, Hagen came across the activity concerning the non-equilibrium steady states (NESS) in quantum many-body systems, popular there at that time. He quickly realised that there was room here for the application of his expertise in the scattering theory. This was the beginning of his fruitful collaboration with J. Rehberg, H. Kaiser and M. Baro, see e.g.: "Macroscopic current induced boundary conditions for Schrödinger-type operators" (*Integral Equ. Oper. Theory* 2003), "Dissipative Schrödinger–Poisson systems" (*J. Math. Phys.* 2004), "A quantum transmitting Schrödinger–Poisson system" (*Rev. Math. Phys*. 2004) and "Classical solutions of drift–diffusion equations for semiconductor devices: The two-dimensional case" (*Nonlinear Analysis* 2009).

At the same time, Hagen never ceased to pay attention to the "purely" mathematical aspect of the NESS and its possible applications, as seen in the papers (with H. Cornean and V. Zagrebnov) "The effect of time-dependent coupling on non-equilibrium steady states" (*Annales Henri Poincaré* 2009), "The Cayley transform applied to non-interacting quantum transport" (*J. Funct. Anal.* 2014) and "A new model for quantum dot light emitting-absorbing devices: proofs and supplements" (*Nanosystems* 2015).

Note that the last two papers were part of the thesis of his Ph.D. student Lukas Wilhelm (WIAS Berlin).

A similar inclination was shown by Hagen in the papers "Non-equilibrium current via geometric scatterers" (*J. of Phys. A* 2014) with P. Exner, M. Tater and V. Zagrebnov and "A model of electron transport through a boson cavity" (*Nanosystems* 2018) with A. Boitsev, J. Brasche and I. Popov. The mathematical background of this model was developed within Hagen's important project on *boundary triplet* technique in the paper "Boundary triplets, tensor products and point contacts to reservoirs" (*Annales Henri Poincaré* 2018) by the same authors including M. Malamud. There the boundary triplet technique was employed. In this paper, a model of electron transport through a quantum dot assisted by a cavity of photons is proposed. In this model, the boundary operator is chosen to be the well-known Jaynes–Cummings operator which is regarded as the Hamiltonian of the quantum dot.

The beginning of the collaboration between Hagen and Mark Malamud dates back to the end of the nineties. Originally it was influenced by Hagen's interest in extensions of a symmetric operator with a gap and his joint results with S. Albeverio and J. Brasche. This collaboration started with an attempt to apply the technique of boundary triplets and the corresponding Weyl functions to the problem of existence (and description) of self-adjoint extensions with prescribed spectrum within a gap. Their result, obtained together with S. Albeverio and J. Brasche, was published in "Inverse spectral theory for symmetric operators with several gaps: scalar type Weyl functions" (*J. Funct. Anal*. 2005).

Mark Malamud and Hagen Neidhardt, Prague (2009)

Later on, Hagen (together with J. Behrndt and M. Malamud) applied the Weyl function technique to investigating scattering matrices of two resolvent comparable self-adjoint operators, i.e., operators with a trace-class resolvent difference. In this direction they published two papers "Scattering matrices and Weyl functions" (*Proc. Lond. Math. Soc.* 2008) and "Scattering matrices and Dirichlet-to-Neumann maps" (*J. Funct. Anal.* 2017). There the scattering matrix of two resolvent comparable self-adjoint extensions of a symmetric (not necessarily densely defined) operator with equal deficiency indices has been expressed by means of the limit values of the Weyl function on the real axis and a boundary operator. The abstract result was then applied to various different realisations of the Schrödinger operator, where the Weyl function is closely related to the classical Dirichlet-to-Neumann map.

In 2007 the assertion of the first paper for the scattering matrix was generalised to the case of a pair of self-adjoint and maximal dissipative extensions of a symmetric operator with finite deficiency indices in "Scattering theory for open quantum systems with finite rank coupling" (*Math. Phys. Anal. Geom.* 2007).

Using the formula for the scattering matrix, the authors recovered a connection, first discovered by V.M. Adamyan and D.Z. Arov, between the Lax–Phillips scattering matrix and the characteristic function of the maximal dissipative operator. Moreover, it was shown there that the Lax–Phillips scattering matrix coincides with the lower diagonal entry of the scattering matrix of the pair of two self-adjoint extensions, with one of them being a minimal self-adjoint dilation of the dissipative operator under consideration.

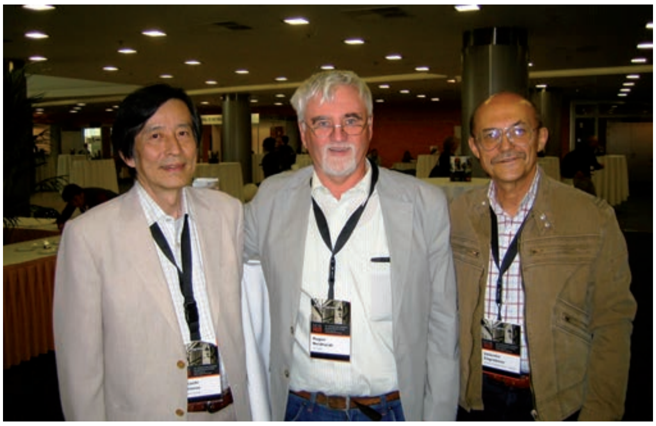
Let us next mention another of Hagen's joint papers with M. Malamud, "Perturbation determinants for singular perturbations" (*Russian J. of Math. Phys*. 2014). Here the boundary triplets technique was applied to perturbation determinants (PD) for pairs of resolvent comparable operators. Treating both operators as proper extensions of a certain symmetric (not necessarily densely defined) operator and choosing a boundary triplet, a PD is expressed via the Weyl function and boundary operators. In applications, it allows one to express a PD of two boundary value problems (BVPs) directly in terms of boundary conditions and the Weyl function. In particular, a PD for two BVPs for Schrödinger operators in a domain with smooth compact boundary via the Dirichlet-to-Neumann map is explicitly computed.

Finally, the series of Hagen's publications in collaboration with M. Malamud and V. Peller deserve a mention: "Trace formulas for additive and non-additive perturbations" (*Advances in Math.* 2015), "Analytic operator Lipschitz functions in the disc and trace formulas for functions of contractions" (*Func. Anal. and Appl.* 2017), and "Absolute continuity of spectral shift" (*J.Funct.Analysis* 2019).

As is clear from their titles, the papers are devoted to the Krein-type trace formulae for pairs of resolvent comparable operators. Here Hagen returned to the subject of his Dr.Sc. dissertation (1987). In particular, it was shown that a pair of contractions (maximal dissipative operators) admits a (non-unique) complex valued summable spectral shift function (SSF), i.e. their resolvent difference outside the unit disc admits a Krein-type representation. Besides, the SSF can be selected to have non-negative (or non-positive) imaginary part whenever the first (second) operator is unitary. A particular case of the later result, where the resolvent difference belongs to the ideal which is slightly narrower than the trace class one and defect operators are of the trace class, was analysed by Hagen (jointly with V.M. Adamyan) in "On the summability of the spectral shift function for pair of contractions and dissipative operators" (*J. Oper. Theory* 1990) .

In the two last papers it was also shown that the maximal class of functions for which the Krein-type trace formula holds is the class of the operator Lipschitz functions, which are analytic in the unit disc.

Note that the problem of description of the maximal class of functions for which the trace formula holds for any pair of self-adjoint operators with trace class dif-



Takashi Ichinose, Hagen Neidhardt, Valentin Zagrebnov, Prague (2009).

ference was posed by M.G. Krein in 1964, and was then solved by V. Peller in 2016. Thus Hagen, jointly with M. Malamud and V. Peller, obtained a solution of the version of the M.G. Krein problem for pairs of contractions (maximal dissipative operators).

Note that Hagen constructed the first example of a pair of contractions which does not admit a real valued locally summable SSF in the paper: "Scattering matrix and spectral shift of the nuclear dissipative scattering theory" (*Operators in indefinite metric spaces, scattering theory and other topics*, Birkhäuser Verlag, Basel 1987). Conversely, in the last paper of the series (*J. Funct. Analysis* 2019) it was proved that a real valued SSF, which is A-integrable (in the sense of A.N. Kolmogorov), always exists.

Note also that in *J. Funct. Analysis* 2019 the authors solved in passing M.S. Birman problem: to find a proof of absolute continuity of a spectral shift measure relied on the theory of double operator integrals. Birman's interest has been inspired by his (joint with M.Solomyak) approach to SSF. The authors' proof is based on S. Nagy-Foias result on ac-continuity of the spectral measure of a minimal unitary dilation of a simple contraction.

Hagen's role as a QMath conference co-organiser was not exhausted by the two events mentioned above. In 2012 he came to rescue when the original plan ran into trouble, and it was his work which made the meeting at the Humboldt University in Berlin possible. As in the previous cases, he co-edited the proceedings volume of QMath12, which was published this time by World Scientific in 2014.

Another of Hagen's projects starting in the same year was the book *Trotter–Kato product formulae*. It was planned for the Springer Lecture Notes in Mathematics series and is actually still in progress.

Farewell in 2016

After his retirement from WIAS in 2016, Hagen was still active and kept a "corner" at the institute to host his visitors and collaborators. With his student Artur Stephan and V. Zagrebnov, Hagen returned to the *evolution semigroup* approach to the construction of *solution operator* (propagator) for the non-autonomous Cauchy problem in Hilbert and Banach spaces. In fact, this idea goes back to his Ph.D. thesis from 1979 about the Howland–Evans–Neidhardt approach to the solution of abstract non-autonomous Cauchy problems. A new aspect was to use the full power of the Trotter product formula approximations for evolution semigroups and their one-to-one correspondence with propagators to produce product formula approximations for the latter.

The one-to-one correspondence allowed control over the rate of convergence of approximants for propagator: "Convergence rate estimates for Trotter product approximations of solution operators for non-autonomous Cauchy problems" (first in archive: arXiv:1612.06147 (2016) and finally in the *Publications of Research Institute for Mathematical Sciences, Kyoto* 2020).

The main results appeared in the series of papers "On convergence rate estimates for approximations of solution operators for linear non-autonomous evolution equations" (*Nanosystems: mathematics* 2017), "Remarks on the operator-norm convergence of the Trotter product formula" (*Integral Equations and Operator Theory* 2018), "Trotter Product Formula and Linear Evolution Equations on Hilbert Spaces" (*Analysis and Operator Theory*, vol.146, 2019).

Beyond Mathematics

Hagen was an extraordinary person.

He loved mathematics as his first priority. When visiting any new place on earth for scientific collaboration, he did not seem all that interested in taking time out to do sightseeing, etc.; what he wanted to do was mathematics. So the colleague who had invited him would have to preach at him to take a break and do something else, otherwise he would just keep sitting in front of his PC and doing mathematics for the duration of his visit.

Second, he loved Nature: he loved hiking, he loved mountains, preferably as high as possible. He enjoyed setting himself a challenge, be it fitness training, mountaineering, or swimming through a dam or very far from the coast of the Mediterranean Sea in Marseille.

Hagen read several daily newspapers every day, and he declared: "A high level of general education is to be inter-

Hagen Neidhardt in Kanazawa (2010).

ested in politics and sports, but also in the small things of everyday life." Hagen had an extraordinary memory for time – for example, he had absolutely no problem recalling key sporting events and the winners. He also liked to watch live football matches at the Berlin stadium.

Hagen was rational and objective, but he could be very funny as well. He had the knack of being able to reverse the mood in "unpleasant" situations by his good, amusing comments, but he could also become emotional and philosophical. He was impressed by the way that time progresses. Hagen enjoyed music, for example classical music by Tchaikovsky, but also modern interpreters such as ABBA. He loved space and everything about space travel and technology; sometimes he had tears in his eyes while watching a rocket launch. But he also found joy in the simple things like the view from the terrace into the greenery. Hagen was a family man. When Hagen and Hiltrud celebrated their silver wedding anniversary, he declared: "Dear Hille, I love you more than my mathematics." Hagen loved his children and always stood by them with support and advice, without ever making empty promises. He very much admired his first granddaughter Sophie. He had started teaching her the digits of π.

Now you can see π on the gravestone of Hagen Neidhardt (20.11.1950–23.03.2019) in the Städtische Friedhof Pankow-Buch.

We are greatly privileged to have been friends and collaborators of Hagen. We miss him so much.