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Numerics and Simulation
Elective subject mathematics

Exercise sheet 2, April 24, 2024

Exercise 6: Create a "L-shaped" domain in Netgen with the corner points $(0,0), (1,0), (1,1), (-1,1), (-1,-1), (0,-1)$ and a related mesh with meshsize 0.2. Assign separate boundary conditions to the lines including the point $(0,0)$ and the other lines. Please check `https://docu.ngsolve.org/latest/i-tutorials/unit-4.1.1-geom2d/geom2d.html#Using-lines-and-splines` for instructions.

Exercise 7: Consider the Dirichlet boundary value problem of the Laplace equation for the "L-shaped" domain of Exercise 6. The predefined solution is given in polar coordinates by $u(r,\varphi) = r^{2/3}\sin(2\varphi/3)$ for $\varphi \in [0, 3\pi/2]$.

a) Implement the solution u, i.e. the transformation to polar coordinates. Be cautious evaluating the Dirichlet data for the discrete extension along the lines including the point $(0,0)$.

b) Refine the mesh several times and create a related table providing the refinement level, the number of vertices (dofs), the errors, and the related experimental orders of convergence (eoc) for linear and quadratic finite elements.

c) Why are the observed orders of convergence in agreement with the theory?

Exercise 8: Implement the explicit Euler method for the lowest order FEM for the initial Dirichlet boundary value problem of the heat equation on the unit square and the time interval $(0, 0.1)$ with the predefined solution $u(x,t) = 16x_1(1-x_1)x_2(1-x_2)\exp(-t)$. Use an $L_2(\Omega)$ projection of the initial datum for the approximation at $t_0 = 0$. You may find some advises at `https://docu.ngsolve.org/latest/i-tutorials/unit-3.1-parabolic/parabolic.html`. Write your own script. Check for convergence at time $t = 0.1$ on a few spatial refinement levels and for appropriate time step sizes.

Exercise 9: Consider the stiffness and the mass matrix of the lowest order FEM for the Laplace equation in the interval $(0,1)$ for uniform meshsize h. Show that the vectors from the lecture are eigenvectors with the eigenvalues (2.1.11) and (2.1.13), respectively.

Exercise 10: Derive two variational equations for the temporal DG discretization (2.1.18) of the considered initial Dirichlet boundary value problem of the heat equations for piecewise linear trial and test functions, i.e. $q = 1$.