Technische Universität Graz

Workshop on

Computational Electromagnetics

Graz, July 12–13, 2010

O. Steinbach (ed.)

Berichte aus dem
Institut für Numerische Mathematik

Book of Abstracts 20010/10

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Technische Universität Graz
Institut für Numerische Mathematik
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Program

Monday, July 12, 2010	
9.30–10.30	L. Demkowicz (Austin) A new class of discontinuous Petrov–Galerkin Finite Element methods with applications to impossible problems I
10.30–11.00	Coffee
11.00–12.00	L. Demkowicz (Austin) A new class of discontinuous Petrov–Galerkin Finite Element methods with applications to impossible problems II
12.00–13.30	Lunch
13.30–14.30	S. Kurz (Tampere) A space–time view on low–frequency electrodynamics
14.30–15.00	M. Wabro (Darmstadt) Research Challenges for Commercial Electromagnetic Simulation Tools
15.00–15.30	Coffee
15.30–16.00	S. Engleder (Graz) Boundary element methods for the eddy current problem
16.00–16.30	L. Weggler (Saarbrücken) hp BEM for electromagnetic scattering
16.30–16.45	Break
16.45–17.15	M. Neumüller (Graz) A DG finite element method for parabolic equations
17.15–17.45	S. Zaglmayr (Graz) Sparsity optimization of H(div)–conforming hp finite elements on simplices
18.30	Dinner
Tuesday, July 13, 2010	
9.00–10.00	L. Demkowicz (Austin) A new class of discontinuous Petrov–Galerkin Finite Element methods with applications to impossible problems III
10.00–10.30	Coffee
10.30–11.30	L. Demkowicz (Austin) A new class of discontinuous Petrov–Galerkin Finite Element methods with applications to impossible problems IV
11.30–13.00	Lunch
13.00–13.30	M. Windisch (Graz) BETI methods for Maxwell equations
13.30–14.00	M. Fleck (Saarbrücken) Discrete electromagnetism using higher order shape functions
14.00–14.30	C. Pechstein (Linz) Weighted Poincaré inequalities and applications in domain decomposition
14.30–15.00	H. Egger (Graz) Adjoint based sampling methods for electromagnetic scattering
15.00–15.30	Coffee

A new class of discontinuous Petrov–Galerkin (DPG) Finite Element (FE) methods with applications to impossible problems

L. Demkowicz
Institute for Computational Engineering and Sciences (ICES)
University of Texas at Austin

The hp-adaptive finite elements combine elements of varying size h and polynomial order p to deliver approximation properties superior to any other discretization methods. The best approximation error converges exponentially fast to zero as a function of number of degrees-of-freedom. The hp methods are thus a natural candidate for singularly perturbed problems experiencing internal or boundary layers like in compressible gas dynamics.

This is the good news. The bad news is that only a small number of variational formulations is stable for hp-discretizations. By the hp-stability we mean a situation where the discretization error can be bounded by the best approximation error times a constant that is independent of both h and p and, ideally, is of order one. To this class belong classical elliptic problems (linear and non-linear), and a large class of wave propagation problems whose discretization is based on hp spaces reproducing the classical exact grad-curl-div sequence. Examples include acoustics, Maxwell, elastodynamics, poroelasticity and various coupled and multiphysics problems.

We will present a new paradigm for constructing discretization schemes for virtually arbitrary systems of linear PDE's that remain stable for arbitrary hp meshes, extending thus dramatically the applicability of hp approximations. We will use convection dominated diffusion as a model problem to present the method and then review a number of applications for which we have collected some numerical experience including:

- 1D Burgers and compressible Navier-Stokes equations (shocks)
- Timoshenko beam and axisymmetric shells (locking, boundary layers)
- 2D linear elasticity
- 1D and 2D wave propagation (polution error control)

The presented methodology incorporates the following features:

The problem of interest is formulated as a system of first order PDE's in the distributional (weak) form, i.e. all derivatives are moved to test functions. We use the DG setting, i.e. the integration by parts is done over individual elements.

As a consequence, the unknowns include not only field variables within elements but also fluxes on interelement boundaries. We do not use the concept of a numerical flux but, instead, treat the fluxes as independent, additional unknowns.

For each trial function corresponding to either field or flux variable, we determine a corresponding optimal test function by solving an auxiliary local problem on one element. The use of optimal test functions guarantees attaining the supremum in the famous inf-sup condition from Babuska-Brezzi theory.

The local problems for determining optimal test functions are solved approximately with an enhanced approximation (a locally enriched mesh).

By selecting right norms for test functions, we can obtain amazing stability properties uniform not only with respect to discretization parameters but also with respect to the perturbation parameter (diffusion constant, Reynolds number, beam or shell thickness, wave number) In other words, the resulting discretization is “robust.” For a detailed presentation on the subject, see [1-7].

References

[1] L. Demkowicz, J. Gopalakrishnan: A Class of Discontinuous Petrov–Galerkin Methods. Part I: The Transport Equation. Comput. Methods Appl. Mech. Engrg., in print. see also ICES Report 2009–12.

[2] L. Demkowicz, J. Gopalakrishnan: A Class of Discontinuous Petrov–Galerkin Methods. Part II: Optimal Test Functions. Num. Methods Part. Diff. Eq., in review, ICES Report 2009–16.

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[6] J. Bramwell, L. Demkowicz, W. Qiu: Solution of Dual–Mixed Elasticity Equations Using AFW Element and DPG. A Comparison. ICES Report 2010–23.

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Adjoint based sampling methods for electromagnetic scattering

H. Egger[1], M. Hanke[2], C. Schneider[2], J. Schöberl[3], S. Zaglmayr[4]
[1]Karl–Franzens Universität Graz
[2]Johannes Gutenberg Universität Mainz, [3]TU Wien, [4]TU Graz

In this talk, we discuss the efficient solution of inverse electromagnetic scattering problems by sampling methods. Such methods have been justified theoretically also for a variety of similar inverse problems, e.g., for electric impedance or diffuse optical tomography.
The sampling method under investigation is based on a factorization of the electromagnetic Calderon operator, which allows to conclude that a point in the computational domain is inside the scatterer, whenever the trace of a certain Green's function is in the range of the measurement operator. For general geometries or background media, such a Green's function is however usually not available.
We will present an equivalent way to compute the numerical range criterion, which avoids the use of the Green's function. Our approach is based on the solution of certain adjoint problems: all information about the background, the geometry and the measurement setup is encoded in a limited number of adjoint fields – one for each detector in the measurement setup. This allows the evaluation of the integrals required in the factorization method simultaneously for all test points z in the domain.
The solution of the adjoint problems can be done in a calibration step, and after this calibration, the sampling method can be realized very fast, i.e., in parts of seconds, even for three dimensional problems.
The viability of the approach is illustrated with numerical examples.

Boundary element methods for the Eddy Current Problem

S. Engleder, O. Steinbach
TU Graz

Magnetic Induction Tomography is a contactless imaging modality, which aims to obtain the conductivity distribution of the human body. The method is based on exciting the body by magnetic induction using an array of transmitting coils to induce eddy currents. A change of the conductivity distribution in the body results in a perturbed magnetic field, which can be measured as a voltage change in the receiving coils. Based on these measurements, the conductivity distribution can be reconstructed by solving an inverse problem.

The forward problem of Magnetic Induction Tomography can be modeled as an eddy current transmission problem in $\mathbb{R}^3$. By using boundary integral equations we can reduce this transmission problem to a problem on the surface of the conducting object.

In this talk we present a boundary element formulation for this eddy current problem and investigate its properties. Furthermore the use of suitable preconditioners and fast boundary element methods will be discussed.

Discrete electromagnetism using higher order shape functions

M. Fleck
Universität des Saarlandes, Saarbrücken

The basic principle of discrete electromagnetism (DEM) is to discretise the operators occuring in the differential form representation of a given PDE. The appearance of the discretised operators depends on the underlying shape functions. While the lowest order version using Whitney elements is well known for its simple structure, a naturalëxtension to higher polynomial degrees has yet to be found. We analyse the discrete electromagnetism using a hierarchical set of higher order shape functions.

A space–time view on low–frequency electrodynamics

S. Kurz, S. Suuriniemi
Tampere University of Technology

In many engineering applications, electromagnetic wave propagation can be neglected. In this case, the full set of Maxwell's equations can be approximated by the well-known magnetoquasistatic (MQS, eddy current) or the electroquasistatic (EQS) model, and we talk about low-frequency electrodynamics. These models are known to be Galilean invariant. If it comes to modelling and simulation of problems involving moving and/or deforming bodies it is important to clearly separate those parts of the theory that are due to a particular choice of the observer from the core of the phenomenon that should be expressed in an observer-independent way. To this end, we give a mathematical model for Galilean space-time, whose key features are affine manifold, absolute time and horizontal Euclidean metric. We show how to formulate MQS and EQS in this space-time model in four dimensions. The usual three plus one-dimensional formulations of MQS and EQS are conceived by an observer-induced decomposition. A change of observer is related to a Galilean transformation of the observed fields. Finally, it is shown that for MQS and EQS the Galilean invariance can be extended from inertial to rigid observers. This justifies the usual analysis of rotating electrical machines from the rotor's point of view.

A DG finite element method for parabolic equations

M. Neumüller, O. Steinbach
TU Graz

The time dependent heat equation will be considered as a model problem. This equation will be discretized in the space time cylinder by using a Discontinuous Galerkin approach. In particular for spatial domains $\Omega \subset \mathbb{R}^3$ we therefore have to decompose the space time cylinder in $\mathbb{R}^4$. For this, a method of decomposing a four dimensional object into pentatopes will be presented. Numerical examples will be given, which show the expected rate of convergence.

Weighted Poincaré inequalities and applications in domain decomposition

C. Pechstein[1], R. Scheichl[2]
[1]Johannes Kepler University Linz, [2]University of Bath

Robust solvers for problems with high-contrast coefficients are currently an important and active area of research. In this talk we present weighted Poincaré inequalities of the form

$$\inf_{c\in\mathbb{R}} \int_\Omega \alpha(x)\,|u(x)-c|^2\,dx \;\le\; C_P\;\mathrm{diam}(\Omega)^2 \int_\Omega \alpha(x)\,|\nabla u(x)|^2\,dx$$

for functions u in $H^1(\Omega)$ or a in suitable discrete subspace. For a certain class of piecewise constant and positive weight functions $\alpha(x)$, we can get the constant C_P independent of the values of α, i. e. of high contrast in α.
As a simple example consider the case where α takes two different values on two connected subregions $\Omega^{(k)}$ of Ω. For this situation we can even give estimates on how C_P depends on the subregions $\Omega^{(k)}$. Generalizations to the multi-subregion case are also possible.
Finally, we give some applications in domain decomposition methods, in particular for FETI type methods. With our inequalities we can show condition number estimates that are robust for certain high contrast coefficients, including cases where the subdomain partitioning does not resolve coefficient jumps.

Research Challenges for Commercial Electromagnetic Simulation Tools

M. Wabro
CST, Darmstadt

At CST AG we develop tools for the simulation of various types of electromagnetic problems. The methodical origin of our products was the finite integration technique (FIT), which aims primarily at high frequency time domain applications. But in recent years we have also put much research and development effort in solving other problem classes (low frequency, electromagnetic compatibility, circuits,...) with a wide range of numerical methods (FEM, BEM, MOR,...).
After a brief introduction to the company we will present main fields of applications and disretization/solution techniques and talk about ongoing research and open problems.

hp BEM for Electromagnetic Scattering

L. Weggler
Universität des Saarlandes, Saarbrücken

The mathematical problem we are going to look at is as follows

$$\begin{aligned} \frac{1}{k}\nabla\times\nabla\times\boldsymbol{u}-k\boldsymbol{u} &= \mathbf{0} && \text{in } \Omega^+\,, \\ \gamma_D^+\boldsymbol{u} &= \boldsymbol{m} && \text{on } \Gamma\,, \\ |\nabla\times\boldsymbol{u}(\boldsymbol{x})\times\frac{\boldsymbol{x}}{|\boldsymbol{x}|}-ik\boldsymbol{u}(\boldsymbol{x})| &= o(\frac{1}{|\boldsymbol{x}|^2}), && |\boldsymbol{x}|\to\infty\,. \end{aligned} \tag{1}$$

Its solution describes the electric field component of an scattered electromagnetic wave propagating in the unbounded domain Ω^+. The existence of a fundamental solution for the differential operator

$$\frac{1}{k}\nabla\times\nabla\times -k\,,\quad k>0$$

yields a representation for $\boldsymbol{u}$ in Ω^+. The so-called representation formula describes the electric field $\boldsymbol{u}$ as an extension of its Dirichlet data $\gamma_D^+\boldsymbol{u}$ and its Neumann data $\gamma_N^+\boldsymbol{u}$. The Neumann data $\gamma_N^+\boldsymbol{u}$, however, is unknown. It is to be found by solving a boundary integral equation on Γ. The Boundary Element Method is an appropriate technique to solve this boundary integral equation numerically.
We would like to present numerical results of a Boundary Element implementation using higher order $H(\mathrm{div}_\Gamma,\Gamma)$-conforming shape function. The code is related to an automatic *hp*-adaptive Finite Element implementation solving $2D$ Maxwell problems, [1]. Our idea has been to extend this code in order to solve $3D$ problems by an *hp*-Boundary Element Method. The power of this new implementation is the usage of the $H^1(\Gamma)$-conforming elements to describe curved surfaces and the $H(\mathrm{div}_\Gamma,\Gamma)$-conforming shape functions up to order nine.

References

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BETI methods for Maxwell equations

O. Steinbach, M. Windisch
TU Graz

In this talk we want to present basic ideas for a Tearing and Interconnecting approach for electromagnetic scattering, using boundary integral equations on the local subdomains. The Tearing and Interconnecting approach is normally used for partial differential equations which lead to elliptic bilinear forms. Nevertheless, C. Farhat introduced the FETI also for the Helmholtz equation (using FEM instead of BEM on the local subdomains), now called FETI–H. In [1] we presented a numerical analysis to use this method with boundary instead of finite elements. In this talk now we describe ideas, how this approach can be used for the even more complicated electromagnetic scattering problem. Instead of standard transmission boundary conditions of Dirichlet and Neumann type we may use Robin type interface conditions, which result in a stable formulation which is robust to possible spurious modes.

References

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Sparsity optimization of H(div)-conforming hp finite elements on simplices

S. Beuchler[1], V. Pillwein[2], S. Zaglmayr[3]
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The main issue of hp-finite element methods are their extremely fast convergence properties with respect to the number of unknowns. But with increasing the polynomial order the density of element matrices as well as the costs of numerical integration gets crucial. On tensor-product elements one can easily overcome these difficulties by constructing a product-based finite element basis exploiting the orthogonality relations of 1d-Legendre-type polynomials. As initially suggested by Dubiner and Karniadakis-Sherwin using the Duffy transformation and Jacobi-type polynomials with adapted weights are the remedy for simplicial elements in case of the scalar function spaces $L_2(\Omega)$ and $H^1(\Omega)$.
In this talk we are concerned with the vector-valued function space H(div) which occurs e.g. in various formulations of fluid mechanics or in mixed formulations of elasticity. A conforming and stable hp-fe discretization first requires normal continuity over element interfaces as well as global exactness in the sense of de Rham. For reasons of stability and parameter-robust preconditioning we rely on a finite element basis providing an explicit splitting of the solenoidal and the non-solenoidal higher-order basis functions. This technique turns out to be a further key tool to extend the techniques of sparsity optimization to H(div)-conforming hp-FEM. We discuss the construction principles of the new fe-basis and the analysis of the sparsity pattern of parameter-dependent div-div problems, for which we also use symbolic summation packages in 3D. We conclude with numerical experiments.

Lecturers

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Erschienene Preprints ab Nummer 2008/1

2008/1	P. Urthaler: Schnelle Auswertung von Volumenpotentialen in der Randelementmethode.
2008/2	O. Steinbach (ed.): Workshop on Numerical Simulation of the Maxwell Equations. Book of Abstracts.
2008/3	G. Of, O. Steinbach, P. Urthaler: Fast Evaluation of Newton Potentials in the Boundary Element Method.
2008/4	U. Langer, O. Steinbach, W. L. Wendland (eds.): 6th Workshop on Fast Boundary Element Methods in Industrial Applications, Book of Abstracts.
2008/5	D. Brunner, G. Of, M. Junge, O. Steinbach, L. Gaul: A Fast BE-FE Coupling Scheme for Partly Immersed Bodies
2009/3	G. Of, O. Steinbach: The All–Floating Boundary Element Tearing and Interconnecting Method.
2009/4	O. Steinbach: A note on the stable coupling of finite and boundary elements.
2009/5	O. Steinbach, M. Windisch: Stable boundary element domain decomposition methods for the Helmholtz equation.
2009/6	G. Of, W. L. Wendland, N. Zorii: On the Numerical Solution of Minimal Energy Problems.
2009/7	U. Langer, O. Steinbach, W. L. Wendland (eds.): 7th Workshop on Fast Boundary Element Methods in Industrial Applications, Book of Abstracts.
2009/8	H. Egger, M. Freiberger, M. Schlottbom: Analysis of Forward and Inverse Models in Fluorescence Optical Tomography.
2009/9	O. Steinbach, M. Windisch: Robust boundary element domain decomposition solvers in acoustics.
2009/10	M. Freiberger, H. Egger, H. Scharfetter: Nonlinear Inversion in Fluorescence Optical Tomography.
2009/11	H. Egger, M. Hanke, C. Schneider, J. Schöberl, S. Zaglmayr: Adjoint Sampling Methods for Electromagnetic Scattering.
2009/12	H. Egger, M.Schlottbom: Analysis and Regularization of Problems in Diffuse Optical Tomography.
2009/13	G. Of, T. X. Phan, O. Steinbach: An energy space finite element approach for elliptic Dirichlet boundary control problems.
2009/14	G. Of, O. Steinbach: Coupled FE/BE Formulations for the Fluid–Structure Interaction.
2010/1	G. Of, T. X. Phan, O. Steinbach: Boundary element methods for Dirichlet boundary control problems.
2010/2	P. D. Ledger, S. Zaglmayr: hp–Finite element simulation of three–dimensional eddy current problems on multiply connected domains.
2010/3	O. Steinbach, P. Urthaler: On the unique solvability of boundary integral equations in poroelasticity.
2010/4	S. Engleder, O. Steinbach: Boundary integral formulations for the forward problem in magnetic induction tomography.
2010/5	Z. Andjelic, G. Of, O. Steinbach, P. Urthaler: Direct and indirect boundary element methods for magnetostatic field problems.
2010/6	M. Neumüller, O. Steinbach: A space–time discontinuous Galerkin method for parabolic initial boundary value problems
2010/7	L. Tchoualag, G. Of, O. Steinbach: Circulant matrix methods for Yukawa type linear elasticity problems.

2010/8 A. Klawonn, U. Langer, L. F. Pavarino, O. Steinbach, O. B. Widlund (eds.): Workshop on Domain Decomposition Solvers for Heterogeneous Field Problems, Book of Abstracts.

2010/9 O. Steinbach, G. Unger: Convergence analysis of a Galerkin boundary element method for the Dirichlet Laplacian eigenvalue problem.